Faculty of Engineering and Faculty of Sciences

Sample Entrance Exam in mathematics with full Answers

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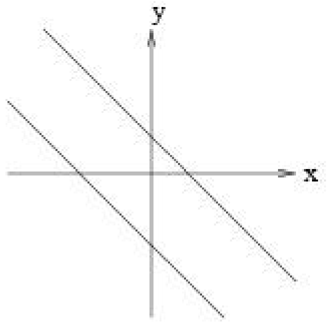
Choose the correct answer for each question:

1- The two parallel lines represent the graph of which of the following of two pairs of equations:



A. $x + y = -3$ and $2x + 2y = 6$.

B. $3x - 2y = 3$ and $2x - 3y = 3$.

C. $x - y = 3$ and $2x - 2y = 6$.

D. $x + 2y = 2$ and $2x + y = 2$.

Solution: Two parallel lines must have the same slope. Hence, for the lines of answer **A** we have

$$y = -3 - x \quad \text{and} \quad y = 3 - x.$$

Thus these two lines have the same slope $a = -1$. For the rest answers **B**, **C** and **D**, the slopes of the two lines are not equal.

The right answer is **A**. ■

2- The function $f(x) = \dfrac{e^{-x^2} - \ln(|x|)}{x - 1}$ has

A. $y = 0$ as an horizontal asymptote in the neighbor of $+\infty$.

B. $x = 1$ as a vertical asymptote.

C. The answers in **A** and **B**.

D. $y = 0$ as an horizontal asymptote in the neighbor of $\pm\infty$ and the answer **B**.

Solution: The domain of definition of the function f is $D_f =]-\infty, 1[\cup]1, +\infty[$. In order to answer to this question, we must test the limit of the function f as $x \to \pm\infty$ and as $x \to 1$. First,

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{e^{-x^2} - \ln(|x|)}{x-1} \qquad (0.1)$$
$$= \lim_{x\to+\infty} \frac{e^{-x^2}}{x-1} - \lim_{x\to+\infty} \frac{\ln(|x|)}{x-1}.$$

But

$$\lim_{x\to+\infty} \frac{e^{-x^2}}{x-1} = \lim_{x\to+\infty} \frac{1}{e^{x^2}(x-1)} = \frac{1}{+\infty} = 0$$

and

$$\lim_{x\to+\infty} \frac{\ln(|x|)}{x-1} = \frac{+\infty}{+\infty} \overset{\text{H.R}}{=} \lim_{x\to+\infty} \frac{1}{x} = \frac{1}{+\infty} = 0.$$

Inserting the last two equations into (0.1), we obtain that $\lim_{x\to+\infty} f(x) = 0$ and therefore $y = 0$ is an horizontal asymptote to f as $x \to +\infty$. Similarly, we show that $y = 0$ is an horizontal asymptote to f as $x \to -\infty$. Next, we check the limit as $x \to 1$:

$$\lim_{x\to 1} f(x) = \lim_{x\to 1} \frac{e^{-x^2} - \ln(|x|)}{x-1} = \frac{e^{-1}}{0} = \infty.$$

Thus $x = 1$ is a vertical asymptote.

The right answer is **D**. ■

3- The derivative of the function $f(x) = x^2 \sin(\ln(x))$ is

A. $f'(x) = \sin(\ln(x)) + x\cos(ln(x))$.

B. $f'(x) = x\cos(\ln(x)) + 2x\sin(\ln(x))$.

C. $f'(x) = \dfrac{\cos(\ln(x))}{x}$.

D. $f'(x) = 2x\sin(\ln(x))$.

Solution: First, we rewrite the function f as below

$$f(x) = u(x)v(x), \quad \text{with} \quad u(x) = x^2 \quad \text{and} \quad v(x) = \sin(\ln(x)).$$

Thus $f'(x) = u'(x)v(x) + v'(x)u(x)$. But $u'(x) = 2x$ and

$$v'(x) = (\sin(\ln(x)))' = (\ln(x))'\cos(\ln(x)) = \frac{\cos(\ln(x))}{x}.$$

Hence

$$\begin{aligned} f'(x) =& u'(x)v(x) + v'(x)u(x) \\ =& 2x\sin(\ln(x)) + x\cos(\ln(x)). \end{aligned}$$

The right answer is **B**. ■

4- The area of the region which lies between the function $f(x) = x^3 e^{x^4}$ and the lines $x = 0$ and $x = 1$ is equal to

A. $e - 1$.

B. $\frac{1}{4}e$.

C. $\frac{1}{4}(e - 1)$.

D. $\frac{1}{4}$.

Solution: The required area is given by the following integral:

$$\mathcal{A} = \int_0^1 f(x)dx = \int_0^1 x^3 e^{x^4} dx.$$

Taking the change of variable $u = x^4$, we get $\frac{du}{4} = x^3 dx$. For $x = 0 \Longrightarrow u = 0$ and for $x = 1 \Longrightarrow u = 1$. It follows that

$$\mathcal{A} = \frac{1}{4}\int_0^1 e^u du = \frac{1}{4}e^u\Big|_0^1 = \frac{1}{4}(e - 1).$$

The right answer is **C**. ■

5- The equation y of the tangent on the curve (C) of a function f at the point $A(x_A, y_A)$ is given by

A. $y = f'(x_A) + y_A$.

B. $y - f'(x_A) = y_A(x - x_A)$.

C. $y = y_A + f(x_A)(x - x_A)$.

D. $y = y_A + f'(x_A)(x - x_A)$.

Solution: The right answer is **D.** ■

6- The distance between the point $(2, 0, 0)$ and the plane $x + 2y + 2z = 0$ is equal to

A. $\frac{1}{3}$.

B. $\frac{2}{3}$.

C. 1.

D. $\frac{4}{3}$.

Solution: The distance from a point $M(a, b, c)$ to a plane $Ax + By + Cz + D = 0$ is given by

$$d = \frac{|a \cdot A + b \cdot B + c \cdot C + D|}{\sqrt{A^2 + B^2 + C^2}}.$$

Thus

$$d = \frac{|2 \times 1 + 0 \times 2 + 0 \times 2 + 0|}{\sqrt{1+4+4}} = \frac{2}{3}.$$

The right answer is **B**. ■

7- The complex number i^{80} is equal to

A. i.

B. $-i$.

C. -1.

D. 1.

Solution: The right answer is **D**. Indeed, $i^{80} = (i^2)^{40} = (-1)^{40} = 1$. ■

8- The real part of the complex number $z = \dfrac{-4+i}{2i-3}$ is equal to

A. $\dfrac{14}{13}$.

B. $\dfrac{13}{14}$.

C. $\dfrac{12}{7}$.

D. $\dfrac{7}{11}$.

Solution: In order to answer this question, we multiply the numerator and the denominator by the complex conjugate of the denominator, and then we simplify:

$$z = \frac{-4+i}{2i-3} = \frac{-4+i}{2i-3} \times \frac{-2i-3}{-2i-3} = \frac{-(-4+i)(2i+3)}{13} = \frac{14}{13} + \frac{5}{13}i.$$

Hence $\Re(z) = \dfrac{14}{13}$.

The right answer is **A**. ■

9- The derivative of the function $f(x) = e^{x(\cos(x)+\sin(x))}$ is

A. $f'(x) = [(x+1)\cos(x) + (1-x)\sin(x)]\, e^{x(\cos(x)+\sin(x))}$.

B. $f'(x) = -(x\cos(x) + x\sin(x))e^{x(\cos(x)+\sin(x))}$.

C. $f'(x) = [-(x+1)\cos(x) + (1-x)\sin(x)]\, e^{x(\cos(x)+\sin(x))}$.

D. $f'(x) = -\dfrac{[x\cos(x) + x\sin(x)]\, e^{x(\cos(x)+\sin(x))}}{\cos(x)}$.

Solution:

$$f(x) = e^{u(x)}, \quad \text{where} \quad u(x) = v(x)w(x), \quad v(x) = x \text{ and } w(x) = \cos(x) + \sin(x).$$

Thus $f'(x) = \left(e^{u(x)}\right)' = u'(x)e^{u(x)}$. But

$$\begin{aligned} u'(x) &= (v(x)w(x))' \\ &= v'(x)w(x) + w'(x)v(x) \\ &= \cos(x) + \sin(x) + (-\sin(x) + \cos(x))x \\ &= (1+x)\cos(x) + (1-x)\sin(x). \end{aligned}$$

Finally

$$\begin{aligned} f'(x) &= u'(x)e^{u(x)} \\ &= \left[(1+x)\cos(x) + (1-x)\sin(x)\right] e^{x(\cos(x)+\sin(x))}. \end{aligned}$$

The right answer is **A**. ■

10- The limit of the function $f(x) = \dfrac{e^{2x}-1}{e^{2x}+1}$ while $x \to +\infty$ is equal to

A. $+\infty$.

B. $-\infty$.

C. -1.

D. 1.

Solution: We know that $\lim\limits_{x\to+\infty} e^{2x} = e^{+\infty} = +\infty$. Thus

$$\begin{aligned} \lim_{x\to+\infty} f(x) &= \lim_{x\to+\infty} \frac{e^{2x}-1}{e^{2x}+1} \\ &= \lim_{x\to+\infty} \frac{e^{2x}\left(1-\frac{1}{e^{2x}}\right)}{e^{2x}\left(1+\frac{1}{e^{2x}}\right)} \\ &= \lim_{x\to+\infty} \frac{\left(1-\frac{1}{e^{2x}}\right)}{\left(1+\frac{1}{e^{2x}}\right)} \\ &= \frac{1-\frac{1}{+\infty}}{1+\frac{1}{+\infty}} \\ &= \frac{1-0}{1+0} \\ &= 1. \end{aligned}$$

The right answer is **D**. ■

11- The limit of the function $f(x) = \dfrac{\ln(x^2+1)}{x}$ while $x \to 0$ is equal to

A. $-\infty$.

B. $+\infty$.

C. 1.

D. 0.

Solution:

$$\lim_{x\to 0} f(x) = \lim_{x\to 0} \frac{\ln(x^2+1)}{x} = \frac{\ln(1)}{0} = \frac{0}{0} \overset{\text{H.R}}{=} \lim_{x\to 0} \frac{(\ln(x^2+1))'}{(x)'}$$
$$= \lim_{x\to 0} \frac{2x}{x^2+1}$$
$$= 0.$$

The right answer is **D**. ■

12- The equation of the plane passing through the point $(3,-1,2)$ and normal to the vector $(-1,2,-3)$ is given by

A. $3(x-1)-(y-2)+2(z+3)=0$.

B. $-3(x+1)+(y-2)-2(z+3)=0$.

C. $(3-x)+2(y+1)+3(2-z)=0$.

D. $-(3+x)-2(y+1)-3(z+2)=0$.

Solution: The equation of the plane passing through a point $A(a,b,c)$ and normal to a vector $\overrightarrow{N}(A,B,C)$ is given by

$$A(x-a)+B(y-b)+C(z-c)=0.$$

Thus, the equation of the plane passing through $(3,-1,2)$ and normal to $(-1,2,-3)$ is given by

$$-(x-3)+2(y+1)-3(z-2)=0 \iff (3-x)+2(y+1)+3(2-z)=0.$$

The right answer is **C**. ■

13- The cross product between the two vectors $(1,1,2)$ and $(-1,1,3)$ is given by

A. $(-1,5,-2)$.

B. $(1,-5,-2)$.

C. $(-1,-5,-2)$.

D. $(\mathbf{1, -5, 2})$.

Solution: The cross product is a vector given by

$$\begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 1 & 1 & 2 \\ -1 & 1 & 3 \end{vmatrix} = (3-2)\overrightarrow{i} - (3+2)\overrightarrow{j} + (1+1)\overrightarrow{k}$$
$$= \overrightarrow{i} - 5\overrightarrow{j} + 2\overrightarrow{k}.$$

Thus the cross product is the vector $(\mathbf{1, -5, 2})$.

The right answer is **D**. ■

14- The equation of the plane passing through the points $(\mathbf{2,2,0})$, $(\mathbf{3,3,2})$ and $(\mathbf{1,3,3})$ is given by

A. $(x-2) - 5(y-2) + 2z = 0$.

B. $(2-x) + 5(y-2) - 2z = 0$.

C. $2x + 2y + z = 0$.

D. $(x-1) + 3(3y-1) + (z+2) = 0$.

Solution: First, we must determine a normal vector to the required plane. Let $A(2,2,0)$, $B(3,3,2)$ and $C(1,3,3)$. Thus $\overrightarrow{AB} = (1,1,2)$ and $\overrightarrow{AC} = (-1,1,3)$ and therefore

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 1 & 1 & 2 \\ -1 & 1 & 3 \end{vmatrix} = (3-2)\overrightarrow{i} - (3+2)\overrightarrow{j} + (1+1)\overrightarrow{k}$$
$$= \overrightarrow{i} - 5\overrightarrow{j} + 2\overrightarrow{k}.$$

Hence, a normal vector on the plane is given by $\overrightarrow{N}(1,-5,2)$. Thus, the plane equation that possibly passing through those three points is one of the plane passing through the point A and normal to $\overrightarrow{N}$ and it is given by

$$(x-2) - 5(y-2) + 2z = 0.$$

The right answer is **A**. ■

15- The equation of the plane passing through the two vectors $(\mathbf{1,1,2})$ and $(\mathbf{-1,1,3})$ is given by

A. $(x-2) - 5(y-2) + 2z = 0$.

B. $(2-x) + 5(y-2) - 2z = 0$.

C. $x - 5y + 2z = 0$.

D. $(x-1) + 3(y-2) + (z-2) = 0$.

Solution: First, we must determine a normal vector to the required plane. Let $\overrightarrow{u}(1,1,2)$ and $\overrightarrow{v}(-1,1,3)$ and therefore

$$\overrightarrow{u} \times \overrightarrow{v} = \begin{vmatrix} \overrightarrow{i} & \overrightarrow{j} & \overrightarrow{k} \\ 1 & 1 & 2 \\ -1 & 1 & 3 \end{vmatrix} = (3-2)\overrightarrow{i} - (3+2)\overrightarrow{j} + (1+1)\overrightarrow{k}$$
$$= \overrightarrow{i} - 5\overrightarrow{j} + 2\overrightarrow{k}.$$

Hence, a normal vector on the plane is given by $\overrightarrow{N}(1,-5,2)$. Thus, the plane equation passing through these two vectors is the one of the plane passing through the origin $O(0,0,0)$ and normal to $\overrightarrow{N}$ and it is given by

$$x - 5y + 2z = 0.$$

The right answer is **C**. ■

16- The trigonometric form of the complex number $1 + i$ is given by

A. $\sqrt{2}\,(\cos(\pi) + i\sin(\pi))$.

B. $\sqrt{2}\left(\cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)\right)$.

C. $\sqrt{2}\,(\cos(\pi) - i\sin(\pi))$.

D. $2\left(\cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3})\right)$.

Solution: The trigonometric form of a complex number z is given by

$$z = |z|\left(\cos(\theta) + i\sin(\theta)\right),$$

where $|z|$ is the modulus of z and θ is its argument.

$$|z| = \sqrt{1+1} = \sqrt{2}.$$

To find the argument θ, we find θ_1 by

$$\tan(\theta_1) = \left|\frac{1}{1}\right| = 1 \Longrightarrow \theta_1 = \tan^{-1}(1) = \frac{\pi}{4}.$$

Since the real part of z, $\Re(z) = 1 > 0$ and the imaginary part of z, $\Im(z) = 1 > 0$, then the argument θ of z is equal to θ_1. Hence

$$z = \sqrt{2}\left(\cos(\frac{\pi}{4}) + i\sin(\frac{\pi}{4})\right).$$

The right answer is **B**. ■

17- The exponential form of the complex number $3(1-i)$ is given by

A. $3\sqrt{2}e^{-\frac{\pi}{4}i}$.

B. $3\sqrt{2}e^{\frac{\pi}{4}i}$.

C. $\sqrt{6}e^{-\frac{\pi}{4}i}$.

D. $\sqrt{6}e^{\frac{\pi}{4}i}$.

Solution: The exponential form of a complex number z is given by

$$z = |z|e^{\theta i},$$

where $|z|$ is the modulus of z and θ is its argument.

$$z = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}.$$

To find the argument θ, we find θ_1 by

$$\tan(\theta_1) = \left|\frac{-1}{1}\right| = 1 \Longrightarrow \theta_1 = \tan^{-1}(1) = \frac{\pi}{4}.$$

Since the real part of z, $\Re(z) = 1 > 0$ and the imaginary part of z, $\Im(z) = -1 < 0$, then the argument θ of z is equal to $-\theta_1 = -\frac{\pi}{4}$. Hence

$$z = 3\sqrt{2}e^{-\frac{\pi}{4}i}.$$

The right answer is **A**. ■

18- If $z = 1 - i$ then z^4 is equal to

A. 4.

B. $2\cos(\pi) + 2i\sin(\pi)$.

C. -4.

D. $4\cos(\pi) - 4i\sin(\pi)$.

Solution: The idea is to determine the Trigonometric form of z and then to take the power four. Equivalently,

$$z^4 = (|z|(\cos(\theta) + i\sin(\theta)))^4 = |z|^4 (\cos(4\theta) + i\sin(4\theta))$$

where $|z|$ is the modulus of z and θ is its argument.

$$|z| = \sqrt{2}$$

and $\theta = -\frac{\pi}{4}$ (similarly to the previous question). Thus

$$z^4 = (\sqrt{2})^4 (\cos(\pi) + i\sin(-\pi)) = -4.$$

The right answer is **C**. ■

19- The derivative of the function $f(x) = \ln(\cos(3x+1)) - e^{\tan(x^2+1)}$ is given by

A. $f'(x) = -3\tan(3x+1) - 2x\sec^2(x^2+1)e^{\tan(x^2+1)}$.

B. $f'(x) = \tan(3x+1) - 2x\sec^2(x^2+1)e^{\tan(x^2+1)}$.

C. $f'(x) = -3\tan(3x+1) + x\sec^2(x^2+1)e^{\tan(x^2+1)}$.

D. $f'(x) = 3\tan(3x+1) + 2x\sec^2(x^2+1)e^{\tan(x^2+1)}$.

Solution:

$$f(x) = u(x) - v(x), \text{ where } u(x) = \ln(\cos(3x+1)) \text{ and } v(x) = e^{\tan(x^2+1)}.$$

Thus $f'(x) = u'(x) - v'(x)$. But

$$u'(x) = (\ln(\cos(3x+1)))' = \frac{(\cos(3x+1))'}{\cos(3x+1)} = \frac{-3\sin(3x+1)}{\cos(3x+1)} = -3\tan(3x+1)$$

and

$$\begin{aligned}\left(e^{\tan(x^2+1)}\right)' &= (\tan(x^2+1))' e^{\tan(x^2+1)} \\ &= (x^2+1)'\sec^2(x^2+1)e^{\tan(x^2+1)} \\ &= 2x\sec^2(x^2+1)e^{\tan(x^2+1)}.\end{aligned}$$

Therefore

$$f'(x) = -3\tan(3x+1) - 2x\sec^2(x^2+1)e^{\tan(x^2+1)}.$$

The right answer is A. ■

20- The primitive $I = \int x^2 \cos(x^3) dx$ is equal to

A. $3\sin(x^3) + C$.

B. $-\frac{1}{3}\sin(x^3) + C$.

C. $\frac{1}{3}\sin(x^3) + C$.

D. $-3\sin(x^3) + C$.

Solution: Taking the change of variable $u = x^3 \Longrightarrow \frac{du}{3} = x^2 dx$. Thus

$$I = \frac{1}{3}\int \cos(u) du = \frac{1}{3}\sin(u) + C = \frac{1}{3}\sin(x^3) + C.$$

The right answer is **C**. ■

21- The angle between the two vectors $\overrightarrow{u}(1,1,0)$ and $\overrightarrow{v}(0,1,0)$ is given by

A. 0.

B. $\frac{\pi}{2}$.

C. $\frac{\pi}{6}$.

D. $\frac{\pi}{4}$.

Solution: The angle between these two vectors is given via

$$\cos(\theta) = \frac{\overrightarrow{u} \cdot \overrightarrow{v}}{\|\overrightarrow{u}\|\|\overrightarrow{v}\|}.$$

where $\cdot$ denotes the dot product between these two vectors and $\| \ \|$ denotes the norm of each vector. Thus

$$\overrightarrow{u} \cdot \overrightarrow{v} = (1)(0) + (1)(1) + (0)(0) = 1, \ \|\overrightarrow{u}\| = \sqrt{2} \text{ and } \|\overrightarrow{v}\| = \sqrt{1} = 1.$$

Hence $\cos(\theta) = \frac{1}{\sqrt{2}} \Longrightarrow \theta = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$.
The right answer is **D**. ■